

EXTRAPOLATION TECHNIQUES OF KILL PROBABILITY DATA

by

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ABSTRACT. A simple formula is described that can be used to extrapolate the kill probability data of NAVORD Report 7019, (NOTS TP 2382), "A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets", by Eldon L. Dunn, with a reasonable degree of accuracy. There are limitations in using the formula, however; and since the formula has an empirical origin, its limitations are discussed at length with the aid of some theory, several graphs, and some numerical examples.



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FOREWORD

This report is intended as a supplement to NAVORD Report 7019, NOTS TP 2382, entitled "A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets", by Eldon L. Dunn. A technique is presented which can be used to extrapolate kill probability data from the Handbook to determine the effectiveness of larger numbers of cluster-delivered bomblets and larger delivery patterns.

The work was carried out at the U. S. Naval Ordnance Test Station under Bureau of Naval Weapons WepTask RMMO42-004/216-1/F008-08-006.

This report is released at the working level for informational purposes only.

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INTRODUCTION

Recent studies to determine the effects of very large numbers of cluster-delivered bomblets and the effects of the relatively large actual delivery patterns have led to a search for a method of extrapolating data which already exists in NAVORD Report 7019, "A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets".¹ The graphs in the Handbook cover the delivery of up to 400 bomblets in a cluster and delivery standard deviations of up to 8 target widths.

In studying the "Handbook", there seemed to be a remarkable similarity between many of the graphs. Further encouragement was received by superimposing one graph upon another. In some cases, the graphs were virtually identical except for a coordinate scale change. The scale changes can be made by a proportional change in various pairs of variables, where all other variables are constant, as in the following algebraic expressions.

$$\left. \begin{aligned} \frac{N^*}{N} &= \left(\frac{\sigma_{F_x}^*}{\sigma_{F_x}} \right)^2 \\ \frac{K_p^*}{K_p} &= \frac{N}{N^*} \\ \frac{K_p^*}{K_p} &= \left(\frac{\sigma_{F_x}^*}{\sigma_{F_x}} \right)^2 \end{aligned} \right\} \text{ when } \frac{\sigma_{R_x}}{\sigma_{F_x}}, \frac{\sigma_{F_y}}{\sigma_{F_x}} \text{ and } P = \text{constant.}$$

where

- N = number of bomblets or rounds in the salvo
- σ_{F_x} = delivery error standard deviation in x-direction
- σ_{F_y} = delivery error standard deviation in y-direction
- σ_{R_x} = bomblet standard deviation in x-direction
- σ_{R_y} = bomblet standard deviation in y-direction
- K_p = probability that a single random hit on the target will result in a kill
- P = over-all kill probability of a delivered cluster weapon

¹ U.S. Naval Ordnance Test Station. A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets, by Eldon L. Dunn. China Lake, Calif., NOTS, 9 December 1959. (NAVORD Report 7019, NOTS TP 2382), UNCLASSIFIED.

* denotes a different set of values for N , K_p , σ_{F_x} , etc.

$\frac{\sigma_{F_y}}{\sigma_{F_x}}$ ratio indicates the extent to which the cluster pattern is elliptical in the ground plane.

The principle obstacle in using these equations is the unknown limits to which they will hold.

Several attempts were made to explain these relationships on a rigorous theoretical basis without any complete success (this approach is receiving further attention at NOTS).

Eventually, a more complete expression was developed which makes a rather versatile extrapolating tool when the user is aware of its limitations. When P and $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ are held constant,

$$\frac{K_p}{\sigma_{F_x}} \frac{N L/W}{\sigma_{F_y}} = \frac{K_p^*}{\sigma_{F_x}^*} \frac{N^* L/W^*}{\sigma_{F_y}^*},$$

where L = target length, W = target width and σ is expressed in target widths. In other words, when

$$\frac{\sigma_{R_x}}{\sigma_{F_x}} \text{ is constant,}$$

$$P \propto \left(\frac{K_p}{\sigma_{F_x}} \frac{N L/W}{\sigma_{F_y}} \right)$$

at least over a limited region.

LIMITED THEORETICAL ASPECTS

Although a complete theoretical analysis using the basic equation is not available at the present time, a few hints on the probable origin of this parameter grouping will be useful in later discussions of the limitations in using the extrapolation formula.

Fortunately it is possible to prove that, when everything else is constant,

$$K_p N = K_p^* N^*$$

is an excellent approximation under certain conditions.

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If the expression

$$\left\{ 1 - [1 - P(x, y)]^N \right\}$$

in the basic equation (see Appendix) is expanded by the binominal series,

$$(1-x)^n = 1 - nx + \frac{n(n-1)x^2}{2!} - \frac{n(n-1)(n-2)x^3}{3!} + \dots + \frac{(-1)^i n! x^i}{(n-i)! i!},$$

and $\left\{ 1 - [1 - K_p P_h]^N \right\}$

$$= N K_p P_h - \frac{N(N-1)(K_p P_h)^2}{2!} + \frac{N(N-1)(N-2)(K_p P_h)^3}{3!} + \dots,$$

$$= N K_p P_h \left(1 - \frac{(N-1)(K_p P_h)}{2!} + \frac{(N-1)(N-2)(K_p P_h)^2}{3!} + \dots \right),$$

where P_h is the single round hit probability defined by

$$\left(\int_A^B \frac{e^{-\frac{u^2}{2}}}{e^{\frac{u^2}{2}}} du \right) \left(\int_C^D \frac{e^{-\frac{v^2}{2}}}{e^{\frac{v^2}{2}}} dv \right)$$

If the expansion in the brackets, which is multiplied by $N K_p P_h$, would cancel in a proportion such that the basic equation reduces to

$$P \propto \frac{N K_p}{(2\pi)^2 \sigma_{F_x} \sigma_{F_y}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (P_h) \exp \left\{ -\frac{1}{2} \left[\frac{x-b_x}{\sigma_{F_x}} \right]^2 + \left[\frac{y-b_y}{\sigma_{F_y}} \right]^2 \right\} dx dy,$$

the approximation that

$$K_p N = K_p^* N^*,$$

when everything else is held constant, is proven.

That this expansion in the brackets does very nearly cancel in a proportion under certain conditions is shown by the following numerical examples.

If $N=1$, it is obvious that

$$\left\{ 1 - [1 - K_p P_h]^N \right\} = K_p P_h.$$

If N is small, but larger than 1, the approximation is quite inaccurate.

For instance, if $N = 2$, $K_p = 0.8$, $P_h = 0.2$ and $N^* = 4$, $K_p^* = 0.4$, $P_h^* = 0.2$,

then

$$\frac{\left\{ 1 - [1 - (0.8)(0.2)]^2 \right\}}{\left\{ 1 - [1 - (0.4)(0.2)]^4 \right\}} = \frac{0.2944}{0.1836} = \frac{2(0.8)}{4(0.4)} (1.037).$$

Obviously the ratio of the bracketed terms is (1.037) instead of 1; so the ratio $\frac{K_p N}{K_p^* N^*} = \text{constant}$ is not true in this region.

However, if

$$\begin{aligned} N &= 50, K_p = 0.8, P_h = 0.2 \\ \text{and } N^* &= 100, K_p^* = 0.4, P_h^* = 0.2, \end{aligned}$$

then

$$\frac{\left\{ 1 - [1 - (0.8)(0.2)]^{50} \right\}}{\left\{ 1 - [1 - (0.4)(0.2)]^{100} \right\}} = \frac{0.9998363}{0.9997608} = \frac{50(0.8)}{100(0.4)} (1.000075).$$

The error of the approximation is only 0.0075%.

If $N = 50$, $K_p = 0.8$, $P_h = 0.2$ and $N^* = 1000$, $K_p^* = 0.04$, $P_h^* = 0.2$,

then

$$\frac{\{1 - [1 - (0.8)(0.2)]^{50}\}}{\{1 - [1 - (0.04)(0.2)]^{1000}\}} = \frac{0.9998363}{0.9996751} = \frac{50(0.8)}{(1000)(0.04)} (1.00016).$$

The error is still only 0.016%.

If $N = 50$, $K_p = 1$, $P_h = 1$

and $N^* = 100$, $K_p^* = 0.5$, $P_h^* = 1$,

then

$$\frac{\{1 - [1 - (1)(1)]^{50}\}}{\{1 - [1 - 0.5(1)]^{100}\}} = \frac{1.00000}{1 - 7.8886(10^{-31})} = \frac{50(1)}{100(0.5)} (1).$$

The accuracy of the approximation is obviously excellent when the initial $K_p P_h \rightarrow 1$

If $N = 50$, $K_p = 0.02$, $P_h = 0.01$

and $N^* = 100$, $K_p^* = 0.01$, $P_h^* = 0.01$,

then

$$\frac{\{1 - [1 - (0.02)(0.01)]^{50}\}}{\{1 - [1 - (0.01)(0.01)]^{100}\}} = \frac{0.009955}{0.00994} = \frac{50(0.02)}{100(0.01)} (1.001).$$

Here again, the approximation error is small. An error of approximately 0.1% occurs.

The accuracy of the approximation is also demonstrated in Fig. 1, 2 and 3 and will be discussed further. It is concluded from the preceding examples that the starting (lower) value of N should be greater than 50 and the starting value of the product $K_p P_h$ or of the quotient $\frac{K_p}{\sigma R}$ should not be extremely small.

Since the validity of the parameter grouping $(N K_p)$ in the approximation

$$P \propto \frac{N K_p}{(2\pi)^2 \sigma F_x \sigma F_y} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (P_h) \exp \left\{ \frac{1}{2} \left[\left(\frac{x-b_x}{\sigma F_x} \right)^2 + \left(\frac{y-b_y}{\sigma F_y} \right)^2 \right] \right\} dx dy$$

has been proven over the region of interest, it is a little easier to speculate as to the origin and validity of the other parameters in

$$\frac{K_p N (L/W)}{\sigma_{F_x} \sigma_{F_y}} = \frac{K_p^* N^* (L/W)^*}{\sigma_{F_x}^* \sigma_{F_y}^*}.$$

It is not difficult to see the origin of $(\sigma_{F_x} \sigma_{F_y})$ if the integral is regarded as a summation of an infinite number of terms--each containing, among other things, the parameter grouping $\left(\frac{K_p N}{\sigma_{F_x} \sigma_{F_y}}\right)$. The degree to which the remaining terms will tend to destroy the proportion cannot be shown except by the graphical examples that follow later, but it is expected that for large values of σ_R and σ_F , the exponential terms in the infinite expansion will tend to approach unity, and the ratio of the expansions for slightly different values of σ_R , σ_F will be nearly 1.

The inclusion of the term LW or L/W when σ is expressed in terms of target widths is due mainly to intuitive considerations. It is meaningless to speak of kill probabilities and delivery errors without reference to the target size. The expression of delivery error in terms of target width would necessitate the inclusion of a term such as L/W in order to make the expression dimensionally consistent.

The adjustment of the target shape, (L/W) , can be expected to be quite limited and heavily dependent on the starting values of σ_{R_x} , σ_{R_y} (or on σ_{F_x} , σ_{F_y} , if $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ and $\frac{\sigma_{R_y}}{\sigma_{F_y}}$ are constant).

The correspondence between the delivery standard deviation and the cluster pattern size or ammunition standard deviation is unique, as is the correspondence between the target size and ammunition standard deviations. The ratio $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ is a parameter in its own right, and the replacement of σ_{R_x} with $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ is one of the most important steps in permitting the handbook graphs to be superimposed with a good fit.

GRAPHICAL TESTS OF THE LIMITS ON THE EXTRAPOLATION FORMULA

The first test of proportional relationship between the various parameters in the extrapolation formula is made with N and K_p . The originally presented extrapolation formula is solved for K_p^* .

$$K_p^* = K_p \left(\frac{N}{N^*} \right) \cdot \left(\frac{L/W}{(L/W)^*} \right) \cdot \left(\frac{\sigma_{F_x}^* \sigma_{F_y}^*}{\sigma_{F_x} \sigma_{F_y}} \right).$$

If $L/W = (L/W)^*$ and $\sigma_{F_x}^* \sigma_{F_y}^* = \sigma_{F_x} \sigma_{F_y}$, the equation reduces to

$$K_p^* = K_p \frac{N}{N^*}.$$

In Fig. 1, two of the graphs of the handbook are plotted one upon the other, with the abscissa reading $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ instead of σ_{R_x} . The graphs

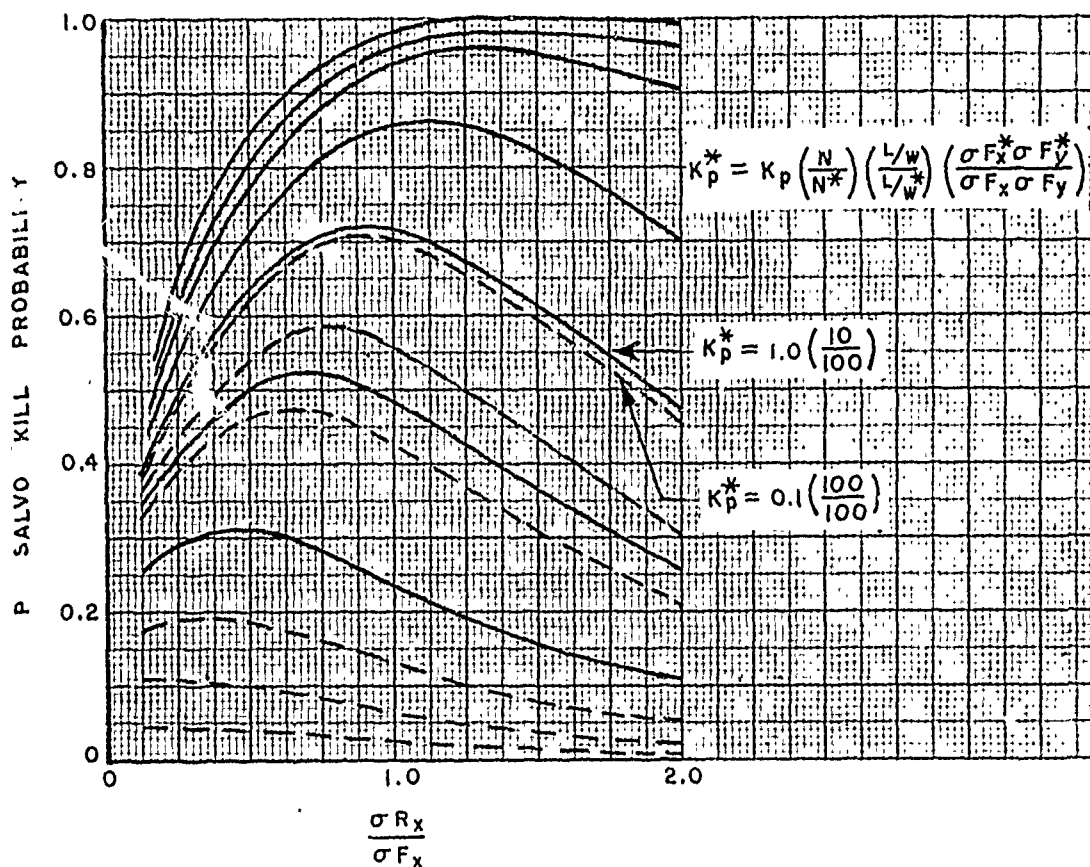


FIG. 1. A Test of the Proportional Relationship Between N and K_p for $\sigma_{F_x} = \sigma_{F_x}^* = 1W$, $L/W = (L/W)^* = 3$, $\sigma_{F_y}/\sigma_{F_x} = 1.5$, When N is Small ($N = 10$).

were selected³ so that

$$\begin{aligned} N_1 &= 10, K_{p1} = 1.0, \\ N_2 &= 100, K_{p2} = 0.1, \\ N^* &= 100, K_{p^*} = 0.1. \end{aligned}$$

A check on one curve will thus be given.

The values of $\sigma_{F_x} = 1W$ and $L/W = 3$ were selected more or less arbitrarily as a start.

From Fig. 1, it is obvious that the two curves that were supposed to match do not match exactly. The curves have a similar shape, but are displaced so that a nearly constant error in P occurs for the various values of $\frac{\sigma_{R_x}}{\sigma_{F_x}}$.

Figure 2 is plotted the same as Fig. 1, except that the lower value on the number of bomblets is changed from $N = 10$ to $N = 50$, and K_p accordingly changed from $K_p = 1.0$ to $K_p = 0.2$. For this plot, there are three curves which can be used as a check on the extrapolation equation.

From Fig. 2, it can be seen that the curves that are supposed to match do come very close to matching over the entire range of

$\frac{\sigma_{R_x}}{\sigma_{F_x}}$ (there are without a doubt some plotting errors in the original data).

To be sure that low values of σ_{F_x} , σ_{F_y} do not have an unexpected effect on the accuracy of the extrapolating proportion, Fig. 3 was plotted. Figure 3 was plotted the same as Fig. 2 except that σ_{F_x} was reduced from $1W$ to $0.5W$. No degradation in accuracy can be detected in Fig. 3 as a result of this change.

It is concluded that as far as can be determined from graphical tests the previous numerical evaluation is correct. In short, the extrapolating proportion

$$K_{p^*} = K_p \frac{N}{N^*}$$

³ Unfortunately, there are only a limited number of handbook parameter combinations which can be used for checking the extrapolation formula. Graphs that are more appropriate could be constructed by interpolating the handbook data; however, this would introduce an error of unknown magnitude which would be undesirable for the present purpose.

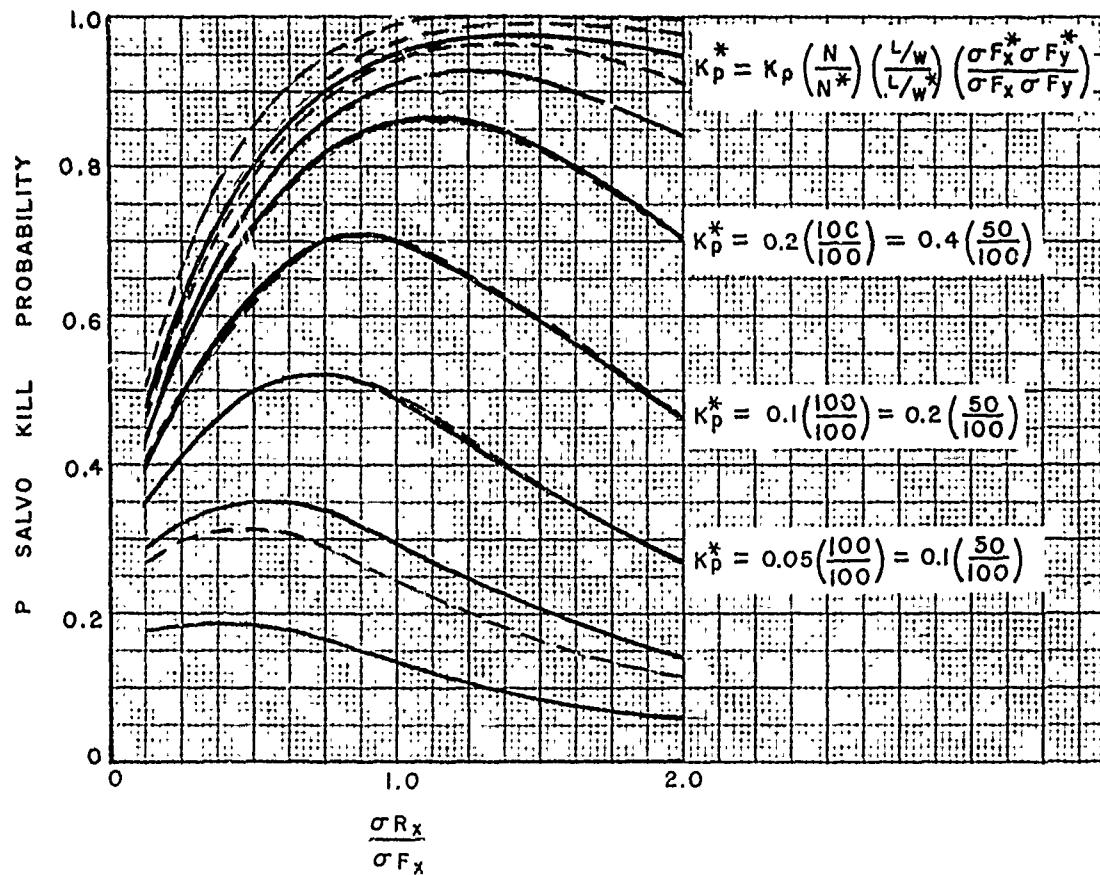


FIG. 2. A Test of the Proportional Relationship Between N and K_p for $\sigma_{F_x} = \sigma_{F_x}^* = 1W$, $L/W = (L/W)^* = 3$, $\sigma_{F_y}/\sigma_{F_x} = 1.5$, When N is Large ($N = 50$).

(when all other parameters are constant) is completely adequate if the lower value of N is large. For most applications the possible limiting region of low $\frac{K_p}{\sigma_R}$ is not expected to be a problem.

Figure 4 is an interesting graph for many reasons. In the first place, it is unique in that the values of σ_{F_x} , σ_{F_y} and N have been chosen so that they have a canceling effect, giving

$$K_p^* = K_p$$

in the extrapolation formula (when $L/W = (L/W)^*$), and

$$K_p^* = K_p \left(\frac{N}{N^*} \right) \left(\frac{L/W}{L/W^*} \right) \left(\frac{\sigma_{F_x}^* \sigma_{F_y}^*}{\sigma_{F_x} \sigma_{F_y}} \right).$$

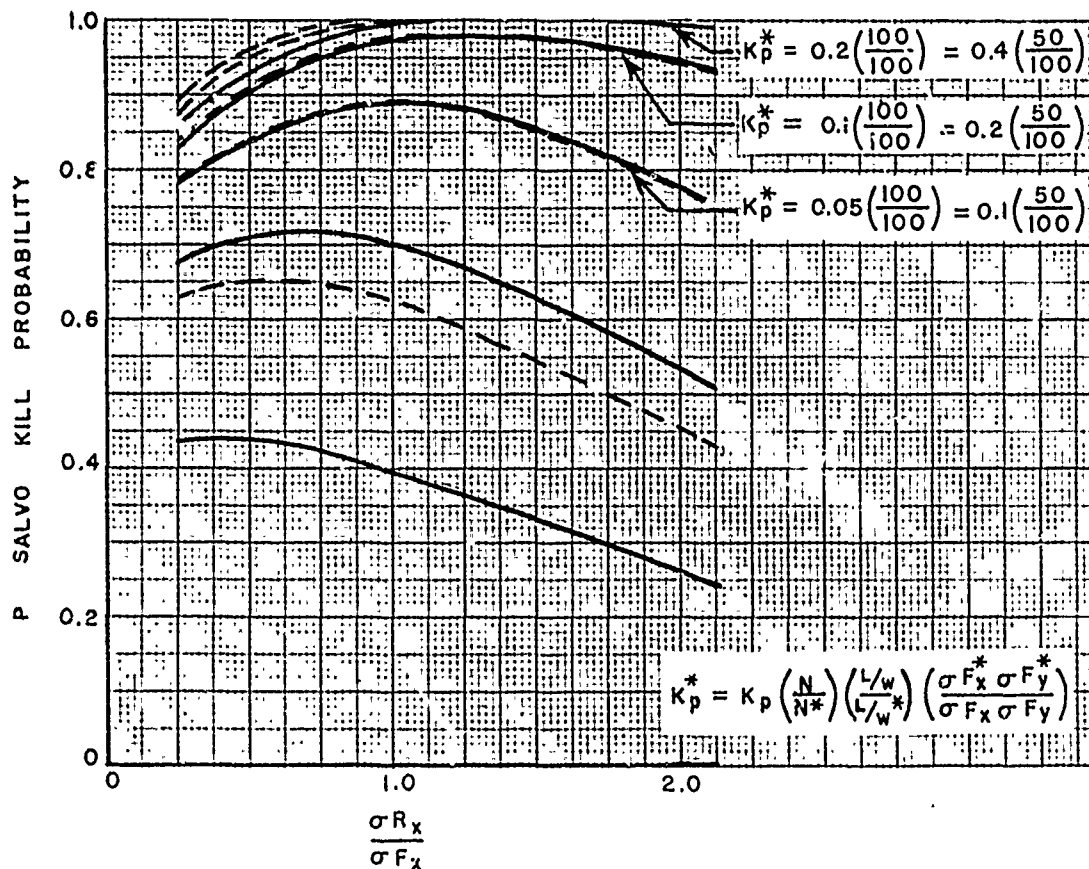


FIG. 3. A Test of the Proportional Relationship Between N and K_p for $\sigma F_x = \sigma F_x^* = 0.5W$, $L/W = (L/W)^* = 3$, $\sigma F_y/\sigma F_x = 1.5$, When N is Large ($N = 50$) and σF_x is Small ($0.5W$).

Figure 4 also shows the importance of the starting value of σF_x that is chosen when extrapolating with σF_x . If the region of interest is to the right of the optimum value of P for the various values of K_p , the starting value is not too critical and $2W$ is apparently adequate. If the region of interest is to the left of optimum P , the starting point is very important. For small extrapolation intervals, a small value of σF_x can be used; but if a large extrapolation is made, the initial value of σF_x should be approximately $8W$. It is recommended that Fig. 4 be used as a guide to point out the regions of limited extrapolation accuracy.

It is to be noted that the data of Fig. 4 for $\sigma F_x = 8W$ and $N = 1600$ are not found in the handbook, because these data are the result of running the IBM Program of the basic equation for a check on the extrapolation technique.

—	K_p^*	FOR TARGET 3	$\sigma F_x = 8W$	$N = 1600$	$\frac{\sigma F_y}{\sigma F_x} = \frac{\sigma F_y^*}{\sigma F_x^*} = 1.5$
- - -	K_p^*	FOR TARGET 3	$\sigma F_x = 4W$	$N = 400$	
- - -	K_p	FOR TARGET 3	$\sigma F_x = 2W$	$N = 100$	

$$K_p^* = K_p \left(\frac{\sigma F_x^*}{\sigma F_y} \right)^2 \left(\frac{N}{N^*} \right) = K_p (i) \text{ IN EACH CASE}$$

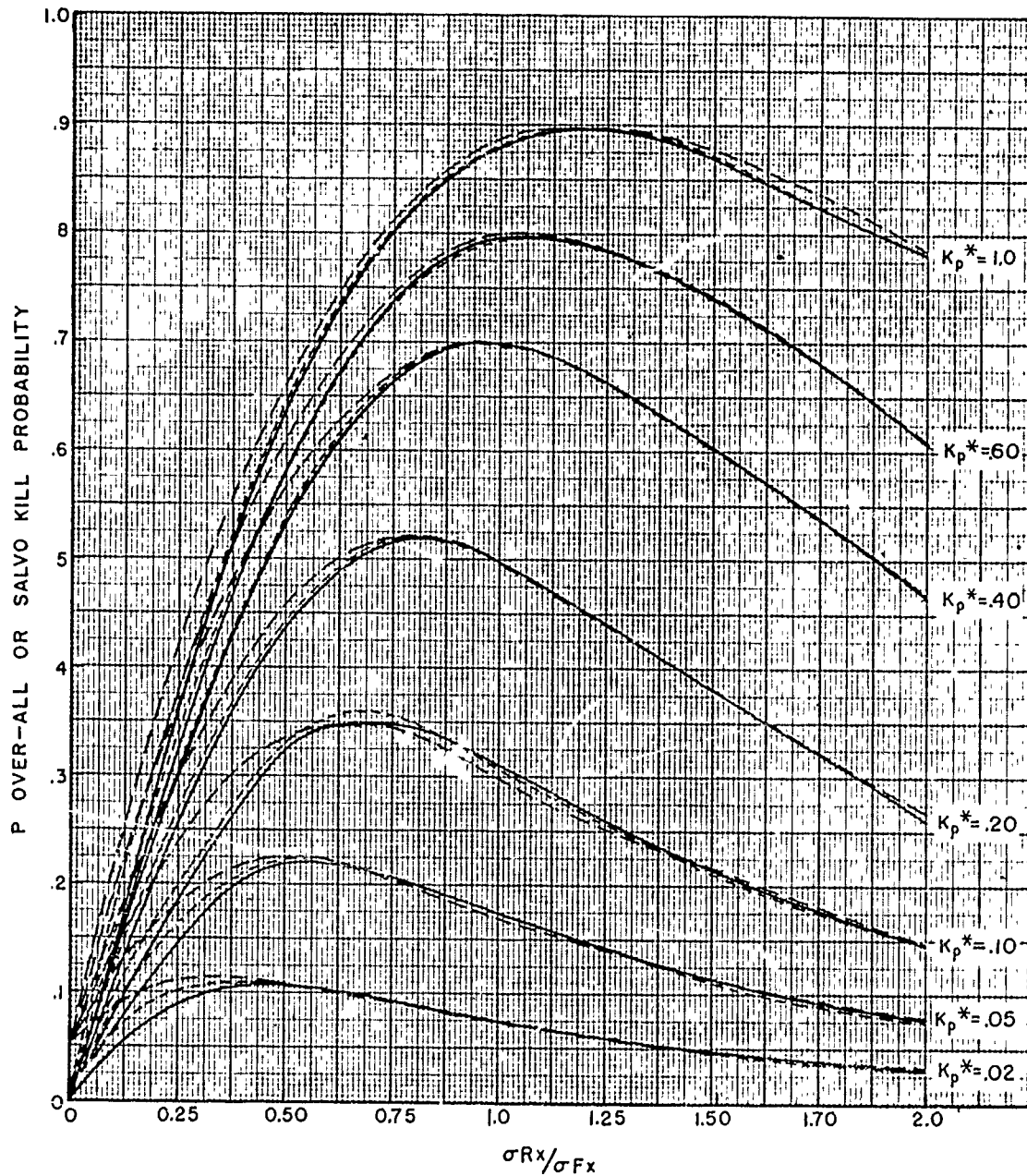


FIG. 4. Reduction of Three Sets of Kill Probability Curves.

Figures 5-8 are intended to show the limitations in using the extrapolation formula to account for the change in the target shape.

Figure 5 shows the handbook graph of $N = 30$, $L/W = 1$ (target 1) superimposed on the handbook graph of $N = 10$, $L/W = 3$ (target 3) for $\sigma_{F_x} = 1W$ in both cases. The canceling effect of N and L/W in the extrapolation formula causes K_p^* to equal K_p . The match is not especially good for this trial value of σ_{F_x} ; so Fig. 6 is constructed.

Figure 6 has identical values of all parameters except for σ_{F_x} , which is increased to $2W$. This time the match is good except where $\frac{\sigma_{R_x}}{\sigma_{F_x}}$ is smaller than that for optimum P .

Figures 7 and 8 are similar to Fig. 5 and 6 except that the target length of one set is increased to $5W$ when $N = 10$, and this is compared to a set which has a target length of $1W$ and $N = 50$.

The match for Fig. 7 is very poor, and for Fig. 8 the match is fair over the portion to the right of the optimum value of P .

It is concluded that a good deal of caution should be exercised in extrapolating to different target shapes. The value of σ_{F_x} should be as large as possible, and each problem should be checked separately for the error in making the extrapolation.

The handbook gives a method for changing the target shape or the dive angle; however, it would have the same limitations as have just been discussed because the two techniques are actually identical.

By changing the symbol definitions

$$\frac{(L/W)^*}{L/W} = \frac{\sigma_{F_x}^* \sigma_{F_y}^*}{\sigma_{F_x} \sigma_{F_y}}$$

to $(L/W)^* = K$ (1, 3 or 5 target widths),

$L/W = L$ (actual length of target in units of target width),

$$\frac{\sigma_{F_y}}{\sigma_{F_x}} = \csc \delta \quad \text{where } \delta = \text{desired dive angle,}$$

and

$$\frac{\sigma_{F_y}^*}{\sigma_{F_x}^*} = 1.5 \quad \text{or } \csc \delta \text{ for the handbook dive angle,}$$

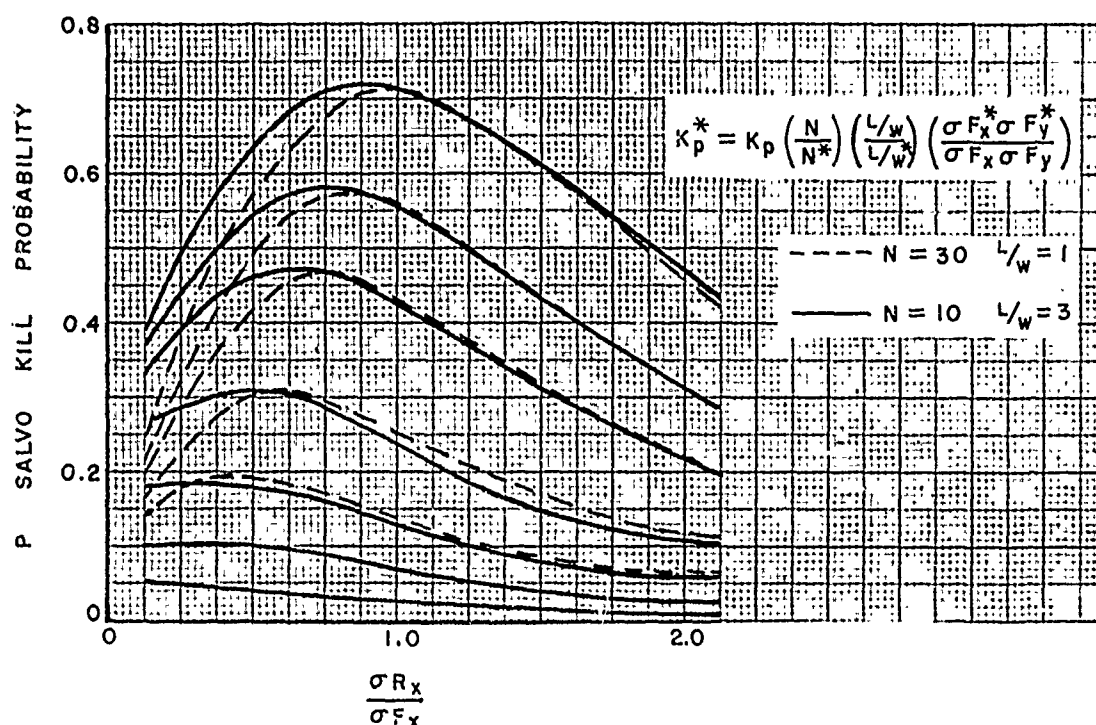


FIG. 5. A Test of the Proportional Relationship Between L/W and N for $\sigma F_x = \sigma F_x^* = 1W$, $\sigma F_y/\sigma F_x = 1.5$, $K_p = K_p^*$.

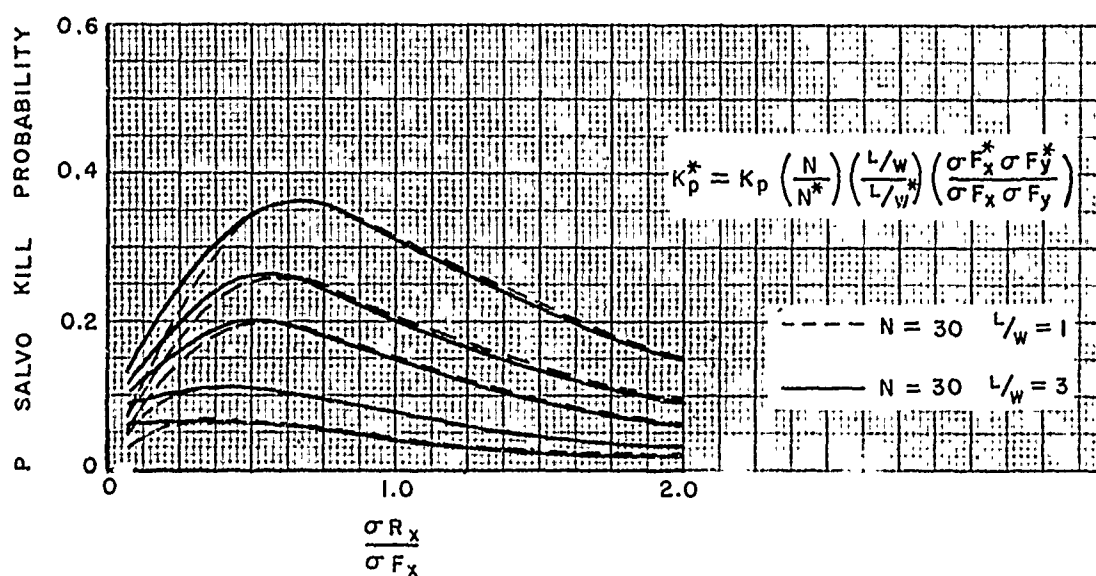


FIG. 6. A Test of the Proportional Relationship Between L/W and N for $\sigma F_x = \sigma F_x^* = 2W$, $\sigma F_y/\sigma F_x = 1.5$, $K_p = K_p^*$.

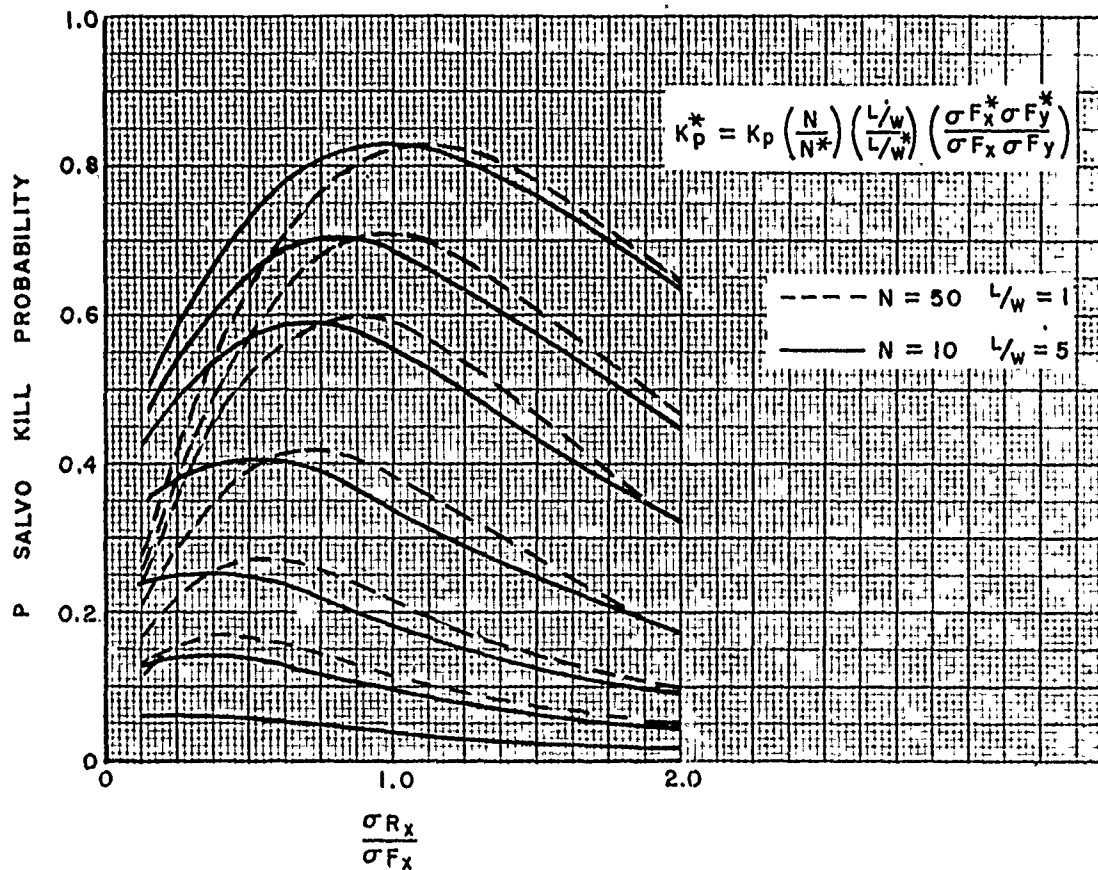


FIG. 7. A Test of the Proportional Relationship Between L/W and N for $\sigma F_x = \sigma F_x^* = 1W$, $\sigma F_y / \sigma F_x = 1.5$, $K_p = K_p^*$.

the result is

$$\frac{K}{L} = \frac{\sigma F_x^{*2} \cdot 1.5}{\sigma F_x^2 \csc \delta}$$

This equation may be written

$$\sigma F_x^* = \sigma F_x \sqrt{\frac{K \csc \delta}{1.5L}},$$

which is the formula that the handbook gives.

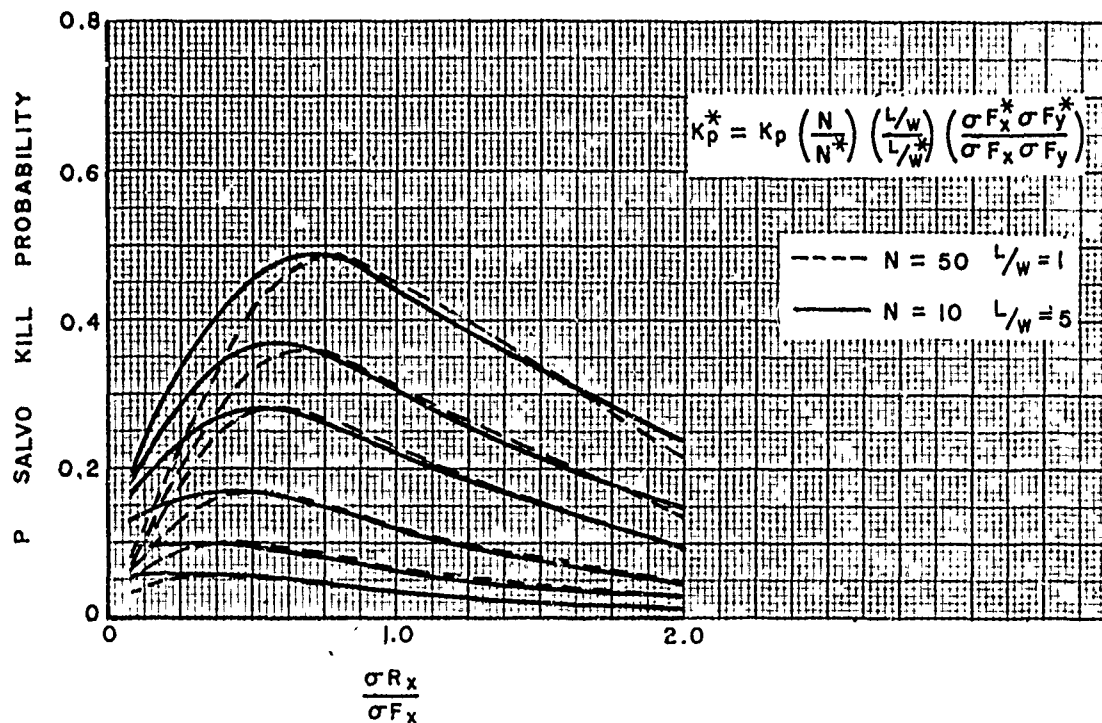


FIG. 8. A Test of the Proportional Relationship Between L/W and N for $\sigma F_x = \sigma F_x^* = 2W$, $\sigma F_y = \sigma F_y^* = 1.5$, $K_p = K_p^*$.

CONCLUSIONS

The relationship

$$\frac{K_p N L/W}{\sigma F_x \sigma F_y} = \frac{K_p^* N^* (L/W)^*}{\sigma F_x^* \sigma F_y^*}$$

when P and $\frac{\sigma R_x}{\sigma F_x}$ are constant, can be used for extrapolating the data of NAVORD Report 7019 quite accurately, if the limitations which are discussed in the text of this report are well understood.

Appendix

BASIC EQUATION

The basic equation, taken from the referenced Handbook (NAVORD Report 7019), is as follows:

$$P = \frac{1}{2\pi\sigma_{F_x}\sigma_{F_y}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - [1 - p(x, y)]^N \right\} \exp \left\{ -\frac{1}{2} \left[\left(\frac{x - b_x}{\sigma_{F_x}} \right)^2 + \left(\frac{y - b_y}{\sigma_{F_y}} \right)^2 \right] \right\} dx dy$$

where

$$p(x, y) = \frac{K_p}{2\pi} \left(\int_A^B e^{-\frac{u^2}{2}} du \right) \left(\int_C^D e^{-\frac{v^2}{2}} dv \right)$$

and

- N = number of rounds in the salvo
- σ_{F_x} = delivery error standard deviation in x-direction
- σ_{F_y} = delivery error standard deviation in y-direction
- b_x = x-coordinate of the aim point of the salvo
- b_y = y-coordinate of the aim point of the salvo
- $A = \frac{X_1 - x}{\sigma_{R_x}}$
- $B = A + \frac{XL}{\sigma_{R_x}}$
- $C = \frac{Y_1 - y}{\sigma_{R_y}}$
- $D = C + \frac{YL}{\sigma_{R_y}}$
- σ_{R_x} = round standard deviation in x-direction
- σ_{R_y} = round standard deviation in y-direction
- X_1 = x-coordinate of lower left corner of the target
- Y_1 = y-coordinate of lower left corner of the target
- XL = length of the target in the x-direction
- YL = length of the target in the y-direction.

ABSTRACT CARD

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